OmniaScience

Journal of Technology and Science Education
JOTSE, 2026 – 16(1): 374-389 – Online ISSN: 2013-6374 – Print ISSN: 2014-5349
https://doi.org/10.3926/jotse.3529

GEOGEBRA APPLETS FOR TEACHING THE DERIVATIVE: CHARACTERISTICS AND INFLUENCE OF THE DERIVATIVE SCHEMA

Luis Dubarbie

Universidad Internacional de La Rioja (Spain)

luis.dubarbie@unir.net

Received May 2025
Accepted January 2026

Abstract

The aim of this work is to identify the main characteristics of GeoGebra resources for teaching the derivative. For this purpose, 76 applets were analyzed according to five variables (derivative schema, mathematical element, representation systems, actions and interactivity) and the influence of the levels of the derivative schema on the rest of the variables was also examined. The data reveal that most applets address the intra- and inter-levels of the derivative schema, and that the graphical and algebraic representation systems are the most frequently used. Furthermore, at the inter-level of the derivative schema, applets address more global properties of the derivative and employ additional representation systems, enabling a greater variety of actions than at the intra-level. However, the results suggest that GeoGebra offers few resources for working at the highest (trans-) level of the derivative schema. The inter-level requires more extensive engagement with the remaining variables, and Fisher's exact test was used to examine the influence of the level of the derivative schema.

Keywords – Derivative, GeoGebra, Derivative schema, Representation systems.

To cite this article:

Dubarbie, L. (2026). GeoGebra applets for teaching the derivative: Characteristics and influence of the derivative schema. *Journal of Technology and Science Education, 16*(1), 374–389. https://doi.org/10.3926/jotse.3529

----

1. Introduction

The concept of the derivative has captured the attention of researchers in mathematics education in recent years. It is a concept inherent to advanced mathematical thinking that generates numerous learning difficulties (Fuentealba et al., 2023; Nurwanhyu et al., 2020; Sánchez-Matamoros et al., 2008). Moreover, the overuse of algebraic procedures contributes to an incomplete understanding of the derivative (Contreras de la Fuente et al., 2003).

Different approaches have been taken in an attempt to address these learning difficulties associated with the concept of the derivative. Among them are the study of students' learning processes and the use of new technologies in the classroom. The first of these approaches is framed within APOS theory, which aims to investigate the process of assimilating the notion of the derivative through the description of different levels of understanding (action, process, object and schema). In fact, this framework has been the focus of numerous studies (Fuentealba et al., 2019; Sánchez-Matamoros et al., 2006), which have attempted to characterize the three levels that make up the derivative schema: intra-, inter- and trans-. With regard to the use of new technologies in mathematics education, the use of GeoGebra particularly stands out (Acikgul & Onuk-Keskin, 2025), highlighting the need for teacher training and the careful supervision of the available resources. For this reason, Barreras et al. (2022) analyzed the GeoGebra applets designed for teaching the concept of the limit of a function, while Yao (2024) studied those that allow students to work on function transformations.

In this study, we aim to analyze GeoGebra applets for teaching the derivative through five variables (mathematical element, derivative schema, representation systems, actions and interactivity), with the following specific objectives:

- To identify the percentage of GeoGebra applets that promote the development of each level (intra-, inter- and trans-) of the derivative schema.
- To determine the proportion of applets associated with each of the categories of the variables studied (mathematical element, representation systems, actions and interactivity).
- To study the influence of the intra- and inter-levels of the derivative schema on the rest of the variables through Fisher's exact test.

2. Theoretical Framework

APOS theory is an adaptation of Piagetian ideas to the framework of advanced mathematical thinking. The aim of this theory is to describe how individuals modify and construct their cognitive structures during the learning of a mathematical concept. To this end, different levels of comprehension (action, process, object and schema) are established, through which individuals progress. The actions, processes and objects are organized into schemas, which, in turn, are structured into three levels: intra-, inter- and trans- (Arnon et al., 2014; Oktaç et al., 2019).

In particular, with respect to the concept of the derivative, this cognitive structure (schema) has frequently been studied (Fuentealba et al., 2017; García et al., 2011; Trigueros et al., 2024). In fact, Sánchez-Matamoros et al. (2006), conducted a study with Baccalaureate and first-year university students involving activities related to the derivative, which allowed the characterization, to a certain degree, of the intra-, inter- and trans-levels of the derivative schema. Special relevance is given to Figure 5 (Sánchez-Matamoros et al., 2006, p. 92), in which the logical relationships among the different mathematical elements that form each of the levels (intra-, inter- and trans-) of the derivative schema are established. This result has subsequently been refined by Fuentealba et al. (2019), who proposed a new characterization of the derivative schema based on a study with university students who had previous instruction in differential calculus.

On the other hand, GeoGebra (Hohenwarter et al., 2009) has become a fundamental tool in the mathematical teaching-learning process in recent years. In fact, in a recent bibliometric study analyzing 301 works from more than 60 countries published in Web of Science-indexed journals between 2009 and 2023 focused on the use of GeoGebra in educational research, Acikgul and Onuk-Keskin (2025, p. 491) state that: "The fact that the number of articles published in the last five years exceeds half of the total, and that these articles have received numerous citations, shows that interest in the use of GeoGebra software in educational research has increased in recent years." To determine specifically how this tool is being used in the classroom, Yohannes and Chen (2021) reviewed articles published between 2010 and 2020, concluding that most studies were conducted at the Secondary Education level, where GeoGebra is most frequently used in mathematical contexts such as geometry and calculus. Furthermore, they found that most studies focused on the analysis of student performance. Numerous works examine the development of students' competences and skills after using GeoGebra in the classroom (García et al., 2021; Romero & García, 2024; Rahman et al., 2025). In addition, the latter study proposed strategies aimed at promoting the proper use of this software by instructors.

In general, the use of new technologies supports the understanding of mathematical concepts within the framework of advanced mathematical thinking, particularly concepts related to calculus. For example, Gavilán-Izquierdo et al. (2021) show that the use of technology in teaching promotes the construction of the derivative concept, as it allows more time for reflection on new content rather than performing excessive calculations. Furthermore, approaching the geometric interpretation of the derivative using technological resources also facilitates learning (Galindo-Illanes et al., 2022). Specifically regarding GeoGebra, Caligaris et al. (2015) present this software as a highly useful tool for illustrating mathematical concepts such as limits, derivatives and integrals, whose graphical representation in textbooks is static and does not facilitate learning. Santos-Trigo et al. (2024) observed that the use of GeoGebra is particularly valuable for learning about the derivative, as it allows the different meaning of this concept to be represented geometrically. Nonetheless, the use of new technologies is not the only resource used to promote understanding of the derivative; APOS theory has also been frequently applied (Font et al., 2016; Fuentealba et al., 2023; Vega-Urquieta et al., 2014).

That being said, the greater use of these technological resources in the classroom also entails the need for enhanced oversight and additional training for educators. In this regard, Barreras et al. (2022) analyzed GeoGebra applets designed for working on the limit of a function. This research examined five variables (limit type, interactivity, conceptual image, representation systems and actions) and concluded that interactivity and the use of multiple representation systems enrich the understanding of limits, as they promote actions across these representation systems, making their presence an indicator of epistemic suitability (Pecharromán, 2013). Reinforcing the need to analyze available GeoGebra resources, Yao (2024) studied applets for working on function transformations, establishing guidelines for their selection and providing recommendations for developing new resources.

Finally, certain obstacles among instructors in using GeoGebra to develop their own applets have been identified, along with the stages of the teaching-learning process in which these applets are implemented (Dubarbie-Fernández et al., 2025).

3. Methodology

In order to analyze GeoGebra applets designed for teaching the derivative, an exploratory and descriptive study was conducted (Leavy, 2017).

3.1. Sample Selection

The applets to be analyzed were selected through intentional sampling (Creswell, 2002), i.e., a non-probabilistic sampling method in which the elements of the sample are chosen because they are relevant for addressing the proposed objectives. Specifically, on September 25, 2023, a search was conducted in the GeoGebra Resources section using the following pathway:

Resources / Materials → Mathematics → Calculus → Diff. calculus → Derivative

Initially, mimicking the process of an instructor searching for resources to enhance teaching of the derivative, the first 100 applets were selected, excluding those that belonged to a group. In the next step, only applets targeting the Secondary Education and Baccalaureate levels were retained, which led to the exclusion of 24 applets. The excluded applets addressed the following content:

Derivative of functions with two variables • Directional derivatives • Partial derivatives • Tangent line in 3D • Tangent plane to a surface
Derivative of a function of a complex variable
Taylor polynomial
Frenet trihedron

Table 1. Contents of the applets not selected

Below is a GeoGebra applet that was discarded because it addresses content inappropriate for the academic level being considered.

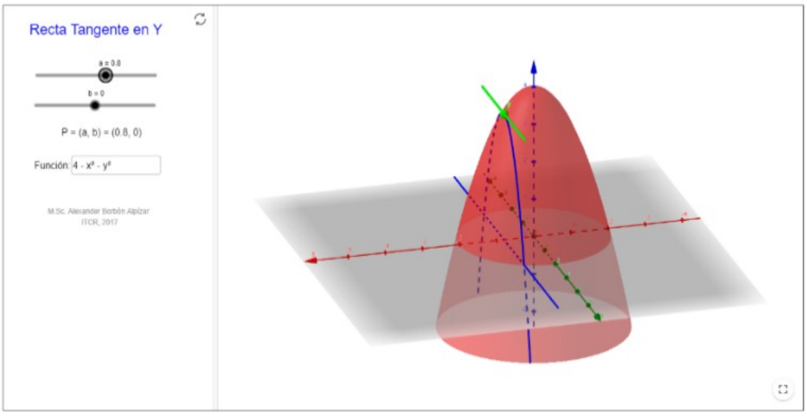

Figure 1. Example of a GeoGebra applet that was not selected (https://www.geogebra.org/m/Wmkqhq3u)

As a result, a total of 76 GeoGebra applets for teaching the derivative in Secondary Education and Baccalaureate studies were ultimately analyzed. Their contents are summarized in Table 2, and are closely related to the mathematical elements mentioned in Figure 5 (Sánchez-Matamoros et al., 2006, p. 92).

Contents	**Quantity**
Geometric interpretation of the derivative at a point (x=a)	33
Derivative operator: rules of differentiation	10
Relationship between f' and f'' and the extrema and inflection points of f (graphical representation)	7
Geometric interpretation of the derivative (global perspective) and the derivative operator (representation of f' given f)	6
Derivative operator (rules of differentiation) and graphical representation of f and f’	4
Derivative at a point as a limit of the mean rate of change	3
Necessary condition for f to be differentiable at x=a (one-sided derivatives)	3
Sufficient condition for f to be an increasing or decreasing function on an interval (a, b)	3
Derivative operator: if the graph of f is a parabola, the graph of f’ is a straight line	1
Inverse of the derivative operator: integration	1
Relationship between the derivative of a function and that of its inverse	1
Applications of the derivative	1
Example of a differentiable function with a discontinuous derivative (geometric interpretation)	1
Lagrange theorem (geometric interpretation)	1
Darboux theorem (geometric interpretation)	1

Table 2. Contents of the applets analyzed

3.2. Data Analysis

The selected GeoGebra applets were analyzed using five variables with a deductive approach, while the influence of the derivative schema level was examined using Fisher's exact test.

The different categories considered in the study were established for all variables, along with the criteria applied for their classification. In particular, for three of the variables (representation systems, actions and interactivity) the structure proposed by Barreras et al. (2022). However, in this study, two new variables are introduced, related to the concept of the derivative and APOS theory, respectively. On the one hand, the mathematical element variable was examined to determine whether the applets facilitate the exploration of derivative characteristics at a pointwise or global level. On the other hand, the applets were classified according to the level (intra-, inter- and trans-) of the derivative schema that they promote among students, based on the results of Sánchez-Matamoros et al. (2006). We chose this characterization of the derivative schema levels because, although it was refined by Fuentealba et al. (2019), the educational level of the students in that study was higher than that considered in this research.

The following table shows the variables, categories and the criteria used for the classification of the selected applets:

Variables	Categories	Criteria
Mathematical element (Sánchez-Matamoros et al., 2006)	Pointwise	Local properties of the derivative are addressed. For example, the geometric interpretation of the derivative of a function at a point x=a.
	Global	The global properties of the derivative are addressed. For example, the study of monotony and the curvature of a function in an interval (a, b).
Derivative schema (Sánchez-Matamoros et al., 2006)	Intra-	Characterized by the identification of an operative action linked to the derivative and the investigation of its internal properties or immediate implications. There is no coordination with other procedures, and difficulties remain in associating it with other properties of the derivative.
	Inter-	The operation associated with the derivative is understood, relevant properties are identified, and it can also be coordinated with similar operations. However, these operations must be associated with contiguous mathematical elements.
	Trans-	Associations are established among the different elements of the derivative schema with minimal restrictions and synthesis is achieved. Global properties not typical of the previous levels are addressed.
Representation systems (Barreras et al., 2022)	Algebraic	The algebraic expression of the function and/or of its derivative is included, and algebraic procedures are performed.
	Graphical	The function and/or an aspect associated with its derivative (geometric interpretation, derivative function, etc.) is represented.
	Numerical	Numerical values are included. For example from points and/or their images, from the value of the derivative at those points, or from the slope of the tangent line to the function, etc.
	Verbal	A mathematic concept related to the derivative is expressed verbally. General descriptions of the activities are excluded.
Actions (Socas, 2007)	Recognition	The derivative object is identified.
	Internal transformation	Transformations are performed (or can be performed) within a single representation system.
	External transformation	The user can perform transformations between different representation systems.
	Coordination	Automatic updating of a representation system when another system is modified.

Variables	Categories	Criteria
Interactivity (Roussou et al., 2006)	Modifying elements	Elements can be modified, such as the function or the point where differentiability is studied.
	Controlling parameters	Parameters that provide relevant information can be modified. For example, the difference quotient in the geometric interpretation of the derivative.
	Allows responses	The user can enter a response and the applet provides feedback.

Table 3. Variables, categories and criteria for the applet classification

4. Results and Analysis

This section is developed with two main focuses of analysis for the selected applets. First, a general descriptive analysis is presented, in which all resources are classified according to the variables and categories described in Table 3. The second part examines the influence of the derivative schema on the remaining variables. For this purpose, the relationships between the derivative schema level and the other variables were analyzed using Fisher's exact test (Agresti, 2002).

4.1. General Analysis

4.1.1. Derivative Schema

To start, the applets were classified according to the level of the derivative schema they promote. To do this, it was determined whether they promote an intra-, inter- or trans-level of knowledge of the derivative schema, based on the descriptions in Table 3 and Figure 5 (Sánchez-Matamoros et al., 2006, p. 92).

The results indicate that more than half of the applets analyzed (51.3%) develop the inter-level of the derivative schema, followed closely by those that develop the intra-level (44.7%). Only 4.0% of the applets promote the trans-level of the derivative schema.

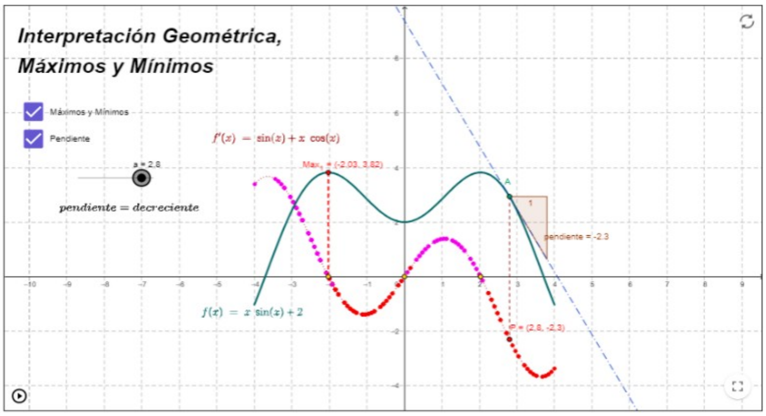

Figure 2. Example of an applet that promotes the inter-level (https://www.geogebra.org/m/zXzreQ2H)

4.1.2. Mathematical Element

Another variable considered in this research is the pointwise or global nature of the mathematical element associated with the derivative in each applet.

In this sample, 55.2% of the resources are oriented towards pointwise mathematical elements, 31.6% illustrate global mathematical elements and 14.5% promote neither local nor global elements. Most of the latter (10 out of 11) are associated with the content *"Derivative operator: rules of differentiation,"* while one examines the content *"Inverse of the differential operator: integration"* (see Table 2).

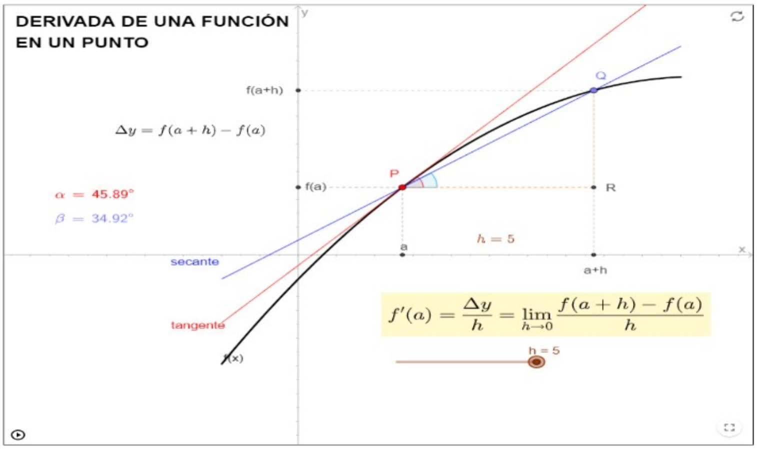

Figure 3. Example of the pointwise applet
(https://www.geogebra.org/m/a3mqNbzD)

Finally, it should be noted that only one applet was identified that addresses both pointwise and global properties of the derivative.

4.1.3. Representation Systems

This section focuses on analyzing the use of representation systems in the applets that form part of the sample. The analysis follows a similar approach to that of Barreras et al. (2022), who studied applets for the limit of a function.

In a preliminary analysis of the presence of each representation system, graphical and algebraic representations are identified in a very high percentage (85.5% and 82.9%, respectively), followed very closely by the numerical representation system (72.6%). The verbal representation system is the least frequent, present in 28.9% of the applets.

The number of representation systems used simultaneously was also analyzed. 14.5% of the applets employ a single representation system, most commonly algebraic, associated with the content *"Derivative operator: rules of differentiation*". The combination of three representation systems occurs most frequently (42.1%), while the presence of two and four systems occurs in similar percentages (23.7% and 19.7%, respectively).

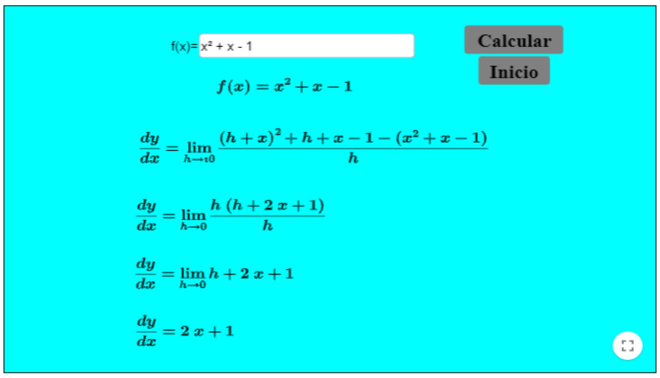

Figure 4. Example of an applet that uses only the algebraic representation system
(https://www.geogebra.org/m/mZdF4eWG)

The following table shows the distribution of applets combining two representation systems.

Representation systems	%
Algebraic and graphical	38.8
Algebraic and verbal	5.6
Graphical and numerical	44.4
Graphical and verbal	11.2
Algebraic and numerical	0.0
Numerical and verbal	0.0

Table 4. Combination of two representation systems

For applets that combine three representation systems, the distribution is shown in the following table:

Representation systems	%
Algebraic, graphical and verbal	6.3
Algebraic, graphical and numerical	87.4
Algebraic, verbal and numerical	0.0
Graphical, verbal and numerical	6.3

Table 5. Combination of three representation systems

Finally, the following figure shows an applet in which all four representation systems are present.

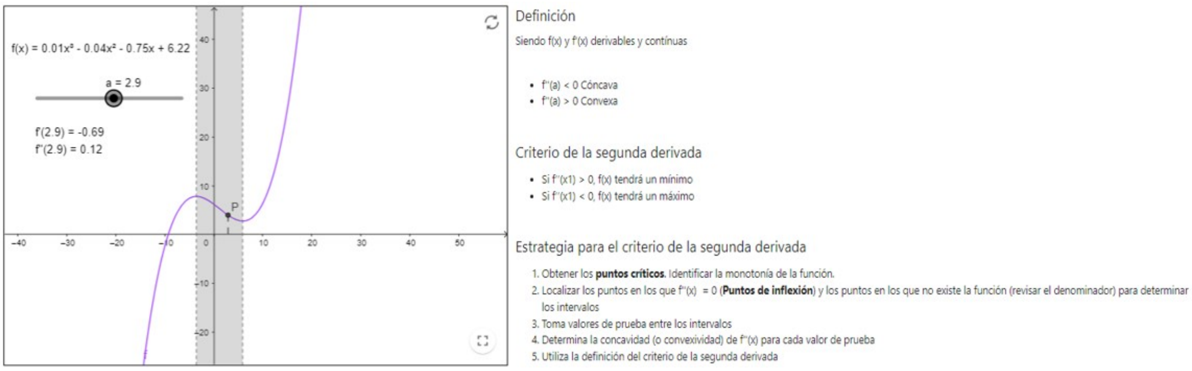

Figure 5. Example of an applet using four representation systems (https://www.geogebra.org/m/yMHwSGmZ)

4.1.4. Actions

Below we analyze the actions carried out between the different representation systems used in the applets. In an initial analysis of this variable, it is observed that all the applets include the identification of some element of the derivative (recognition). On the other hand, the proportion of resources in which internal transformations are performed within the same representation system and in which different representation systems are coordinated is identical (77.6%). Finally, no applet was identified in which external transformations (or conversions) are carried out between representation systems. From these data, the following conclusions can be drawn:

- The only action that is carried out in isolation is recognition, which occurs in 14.5% of the applets analyzed.
- The four actions are never combined in a single applet.
- The combination of recognition, internal transformations and coordination occurs in 67.1% of the resources analyzed in this study.

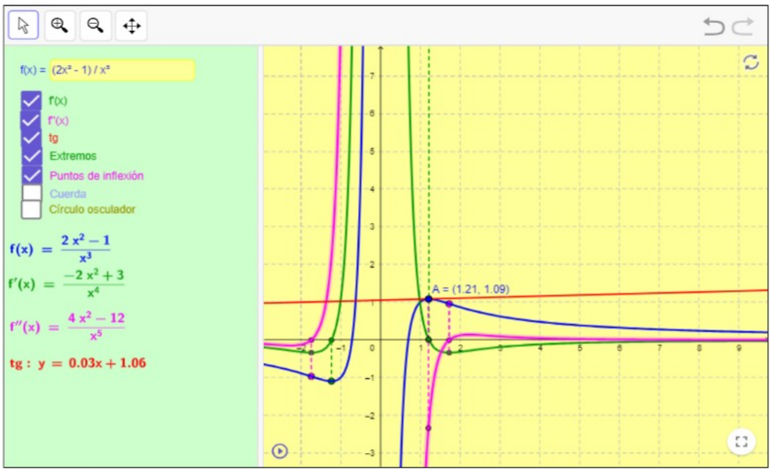

Figure 6. Example of an applet in which three actions are combined
(https://www.geogebra.org/m/RF6XUG5U)

4.1.5. Interactivity

The final aspect analyzed is interactivity, for which the approach proposed by Roussou et al. (2006, p. 2) is followed. Initially, 93.4% of the applets were found to be interactive, while the remaining applets are non-interactive.

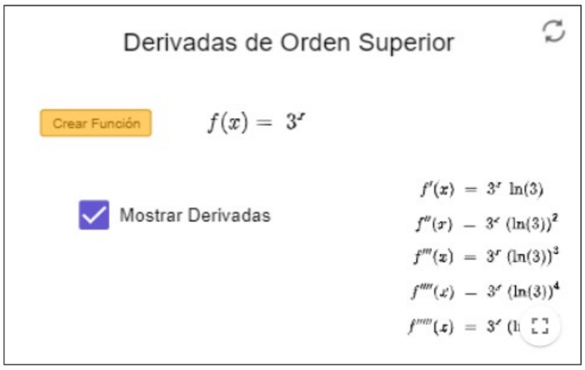

Figure 7. Example of a non-interactive applet
(https://www.geogebra.org/m/vtRhhaag)

Below are the results obtained for different aspects of interactivity, as considered in Table 3.

Interactivity	%
Modifying elements	76.1
Controlling parameters	52.1
Allows responses	9.9

Table 6. Analysis of interactivity

It should be stressed that this latter aspect of interactivity (allows responses) always emerges in isolation and is associated with the content category "*Derivative operator: rules of differentiation*". Furthermore, the applets that implement a single aspect related to interactivity represent 62% of the total, while the rest implement two aspects (modifying elements and controlling parameters).

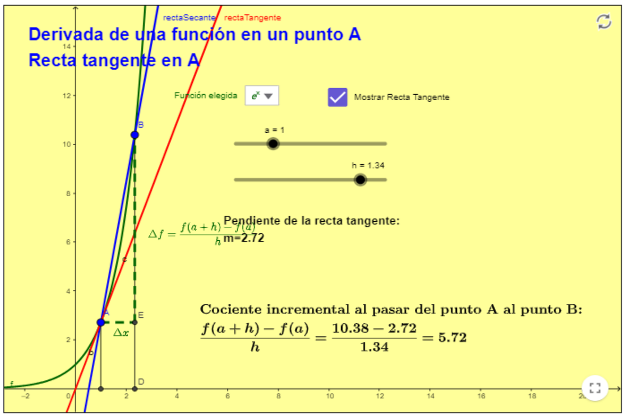

Figure 8. Example of an interactive applet (modifying elements and controlling parameters) (https://www.geogebra.org/m/DMUVEm5P)

4.2. The Influence of the Level of the Derivative Schema

Based on the previously obtained data, the distribution of the applets according to the level of derivative schema promoted is as follows: intra- (34/76), inter- (39/76) and trans- (3/76). With the samples available from the intra- and inter-levels, we proceed to study the influence that these derivative schema levels have on the rest of the variables considered.

4.2.1. Mathematical Element

First of all, we observe how the analysis of pointwise and global properties varies according to the level of the derivative schema.

Variable	Categories	Intra- (%)	Inter- (%)
Mathematical element	Pointwise	61.8	51.3
	Global	5.9	48.7

Table 7. Mathematic element variable according to the level of the derivative schema

In the case of those applets in which the intra-level of the derivative schema is promoted, a clear predominance of pointwise properties is evident. In contrast, in the applets that promote the inter-level, the distribution between pointwise and global properties is more balanced. In addition, when global properties are addressed, the percentage is far higher in the applets that develop the inter-level. Fisher's exact test confirms the existence of a statistically significant dependence between these variables at any level of significance.

4.2.2. Representation Systems

Analyzing the presence of the different representation systems, it is observed that the algebraic and graphical systems constitute the majority of the systems used in the applets that promote both levels of derivative schema, together with the numerical schema in the inter-level (Table 8). Comparing the results obtained at each level of derivative schema, it can be seen that the graphical and numerical systems have a greater presence in the applets that promote the inter-level.

Variable	Categories	Intra- (%)	Inter- (%)
Representation systems	Algebraic	82.4	82.1
	Graphical	70.6	97.4
	Numerical	50.0	82.1
	Verbal	26.5	25.6

Table 8. Representation systems variable according to the levels of the derivative schema

If we now study the number of representation systems for each level of the derivative schema, it is observed that, in both cases, the highest percentage corresponds to the combination of three representation systems (Table 9). On the one hand, a noticeable difference appears in the case of the use of a single representation system.

Variable	Quantity	Intra- (%)	Inter- (%)
Representation systems	1	29.4	5.1
	2	23.5	23.1
	3	35.3	51.3
	4	11.8	20.5

Table 9. Number of representation systems according to the levels of the derivative schema

In this case, Fisher's exact test does not show statistically significant evidence of a relationship between the representation system variable and the level of the derivative schema. However, regarding the number of representation systems, this test confirms the existence of a statistically significant association, with a significance level of 96%.

4.2.3. Actions

Initially, let us examine the influence that the level of the derivative schema has on the different categories of the actions variable.

Variable	Categories	Intra- (%)	Inter- (%)
Actions	Recognition	100.0	100.0
	Internal transformation	61.8	92.3
	External transformation	0.0	0.0
	Coordination	61.8	92.3

Table 10. Actions variable according to the level of the derivative schema

The previous table indicates that, in applets targeting the inter-level of the derivative schema, actions related to internal transformations and coordination are markedly more prevalent.

Variable	Quantity	Intra- (%)	Inter- (%)
Actions	1	26.5	2.6
	2	23.5	12.8
	3	50.0	84.6
	4	0.0	0.0

Table 11. Number of actions according to the level of the derivative schema

These data allow us to confirm that the combination of three actions is the most frequently used at both levels of the derivative schema. On the other hand, it can also be seen that the percentage of applets in which only one action (recognition) is performed is far higher at the intra-level, while the implementation of three actions has a greater presence at the inter-level.

Finally, similarly to what occurs with the representation systems, the actions variable does not show a statistically significant dependence on the levels of the derivative schema according to Fisher's exact test, while the number of actions shows a statistically significant association at the 99% level.

4.2.4. Interactivity

Among the interactive applets, there is the following distribution of the different aspects analyzed, according to the level of the derivative schema.

Variable	Categories	Intra- (%)	Inter- (%)
Interactivity	Modifying elements	59.4	89.2
	Controlling parameters	53.1	51.4
	Allows responses	18.8	2.7

Table 12. Interactivity variable according to the level of the derivative schema

It is observed that, at both levels, the modification of elements is the category of the interactivity variable with the highest percentage. Furthermore, it is interesting to note that the applets that promote the inter-level allow elements to be modified in a higher proportion, although this trend is reversed when considering the applets that allow responses. Finally, Fisher's exact test identifies a statistically significant dependence between the interactivity variable and levels of the derivative schema, with a 94% level of significance.

5. Discussion and Conclusions

Taking into account the characterization of the levels of the derivative schema obtained by Sánchez-Matamoros et al. (2006), it was possible to verify that the large majority of the GeoGebra applets consulted are designed to address the intra- and inter-levels. Thus, it can be inferred that GeoGebra does not offer a large number of resources aimed at addressing the highest level of comprehension (trans-) of the derivative schema. It is therefore not surprising that more than half of the applets address pointwise properties of the derivative and only about a third focus on global properties. However, one of the most relevant aspects in relation to the mathematical element variable is that 14.5% of the applets analyzed are devoted exclusively to the algebraic treatment of the derivative (without addressing either pointwise or global properties). The use of such resources fosters only a partial understanding of this mathematical concept among students (Contreras de la Fuente et al., 2003). This, in turn, adds to the body of research that highlights the tendency of instructors to focus on the algebraic treatment of the derivative, as observed among teachers in training (Vargas et al., 2020) and in studies concerning the differentiability of a function at a point (Dubarbie & García-Gallo, 2023).

On the other hand, the predominant representation systems are the graphical and algebraic systems, followed closely by the numerical system, while the verbal system is clearly the least used. In addition, the combination of three representation systems in a single applet is the most common, which contributes to enriching the derivative schema constructed by the students (Gavilán-Izquierdo et al., 2021; Sánchez-Matamoros et al., 2008). Finally, in those applets in which a single representation system is used, the algebraic system is primarily associated with the content *"Derivative operator: rules of differentiation"*.

With regard to the actions described by Socas (2007) that can be performed between the different representation systems used for the treatment of the derivative, it is noteworthy that none of the applets examined implement external transformations (or conversions). This finding that had also been observed previously in GeoGebra applets designed to explore the limit of a function (Barreras et al., 2022). As occurs in teaching about limits (Ward et al., 2013), this reveals that these types of actions between representation systems generate the greatest difficulties in the teaching of the derivative. Furthermore, in a substantial proportion (67.1%) of the cases, the simultaneous presence of the other three actions (recognition, internal transformation and coordination) was observed across the representation systems employed, indicating the epistemic appropriateness of these resources for teaching the derivative (Pecharromán, 2013).

Unlike the case of applets used for teaching limits (Barreras et al., 2022), the vast majority of those analyzed in this work are interactive (93.4%). Among the interactivity-related aspects considered, the limited presence of applets that allow users to provide a response and subsequently receive feedback is particularly noteworthy. All of these are associated with the algebraic representation system and the content area *"Derivative operator: rules of differentiation"*.

It is therefore recommended that mathematics instructors who use GeoGebra to teach the derivative select applets in which the largest possible number of representation systems is present, while avoiding those in which only the algebraic system is used. In this way, the presence of a greater number of actions between the representation systems is promoted, thereby contributing to ensuring the epistemic suitability of the selected resources. On the other hand, it is also necessary to design applets that promote the development of the trans-level in students, which is characterized by a deep and comprehensive understanding of the properties of the derivative, as well as the ability to establish coherent relationships among the various aspects related to this mathematical concept.

Regarding the influence of the derivative schema level, it has been descriptively observed that addressing a higher level of derivative schema knowledge (inter-level) requires a more extensive use of the other variables considered. For instance, with respect to the mathematical element variable, the primary difference concerns the number of global properties of the derivative that are considered. With regard to the representation systems, the notable difference lies in the number of systems used simultaneously at each level of the derivative schema. In fact, it has been observed that at the intra-level the percentage of applets that present a single representation system of the derivative is far greater, while the opposite occurs in the case of the simultaneous use of three or four representation systems. As a result, the combination of three actions between the representation systems of the derivative occurs more frequently at the inter-level, whereas the presence of isolated actions is found almost exclusively at the intra-level. Lastly, with respect to the interactivity variable, it can be seen that the applets that allow responses are found primarily in the intra-level, and they are associated with resources in which the derivative is treated exclusively from an algebraic perspective. Finally, Fisher's exact test has allowed us to conclude that the variables mathematic element, number of representation systems, number of actions and interactivity depend in a statistically significant manner on the level of the derivative schema, which reinforces the influence that this variable has on the rest of the variables considered.

6. Recommendations

Based on the results obtained in this research, various lines of work emerge. Specifically, given the limited number of GeoGebra applets that support the exploration of the trans-level of the derivative schema, it is recommended that teacher training be promoted with a focus on the design and development of GeoGebra resources that foster the highest level of understanding of this mathematical concept.

Likewise, in today's digital age, the growing presence of artificial intelligence across different areas of the teaching and learning process cannot be ignored. Therefore, further research is recommended on its potential for generating resources that facilitate work at the trans-level of the derivative schema. In fact, numerous studies have examined the applications of ChatGPT in the field of education, as demonstrated by the bibliographic review carried out by Ipek et al. (2023), as well as by studies on the development of calculus concepts using this tool (Urhan et al., 2024). In addition, comparative studies could also be conducted on the use of GeoGebra and other tools commonly used in mathematics teaching, such as Desmos.

Finally, another aspect that has received less attention, but is equally important, is the design of educational situations that promote the development of the derivative schema through the use of resources offered by GeoGebra. Furthermore, their implementation in the classroom would allow cognitive variables to be incorporated into studies on the learning of the derivative using GeoGebra.

Acknowledgement

The author would like to thank the reviewers for their suggestions for improvement, as well as Professor Antonio M. Oller Marcén for his valuable contributions. In addition, this work has been partially funded by the UNIR2024 internal project "Professional development of teachers to promote the optimal integration of simulations and virtual laboratories in the STEM area," awarded to the research group in Mathematics and Experimental Science Education (DIMACE) at the International University of La Rioja.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author received financial support from the International University of La Rioja through the research project "Professional development of teachers to promote the optimal integration of simulations and virtual laboratories in the STEM area," awarded to the research group in Mathematics and Experimental Science Education (DIMACE) at the International University of La Rioja.

Authors' contributions

The author confirms sole responsibility for the following: study conception and design, data collection, analysis and interpretation of results, and manuscript preparation.

Data availability

Data included in the article itself or supplementary material.

Use of Artificial Intelligence

The author declare that the content of the article has not been developed using Artificial Intelligence.

References

Acikgul, K., & Onuk-Keskin, K. (2025). GeoGebra in educational research: A bibliometric analysis. *International Journal of Education in Mathematics, Science, and Technology, 13*(2), 481–497. https://doi.org/10.46328/ijemst.4620

Agresti, A. (2002). *Categorical Data Analysis.* John Wiley & Sons.

Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Fuentes, S. R., Trigueros, M., & Weller, K. (2014). *APOS theory: A framework for research and curriculum development in mathematics education*. Springer. https://doi.org/10.1007/978-1-4614-7966-6

Barreras, A., Dubarbie, L., & Oller-Marcén, A. M. (2022). Análisis de *applets* de GeoGebra para la enseñanza del límite de una función. *Bordón. Revista de pedagogía*, *74*(4), 65–83. https://doi.org/10.13042/Bordon.2022.93361

Caligaris, M. G., Schivo, M. E., & Romiti, M. R. (2015). Calculus & GeoGebra, an interesting partnership. *Procedia-Social and Behavioral Sciences*, *174*, 1183–1188. https://doi.org/10.1016/j.sbspro.2015.01.735

Contreras de la Fuente, A., Luque-Cañada, L., & Ordoñez-Cañada, L. (2003). Una perspectiva de la enseñanza-aprendizaje de la continuidad y la derivada de una función en Bachillerato y Universidad. *Revista de Educación, 331*, 399–419.

Creswell, J. W. (2002). *Educational research: planning, conducting, and evaluating quantitative and qualitative research*. Pearson.

Dubarbie, L., & García-Gallo, A. (2023). La enseñanza de la derivabilidad de una función en un punto. Sistemas de representación y conocimiento especializado del contenido. *UNIÓN – Revista Iberoamericana de Educación Matemática*, *19*(68). https://www.revistaunion.org/index.php/UNION/article/view/1436

Dubarbie-Fernández, L., Barreras, A, & Oller-Marcén, A. M. (2025). El uso de GeoGebra en la enseñanza de conceptos matemáticos: prácticas, barreras y percepciones docentes. *Revista Latinoamericana de Tecnología Educativa - RELATEC*, *24*(1), 77–100. https://doi.org/10.17398/1695-288X.24.1.77

Font, V., Trigueros, M., Badillo, E, & Rubio, N. (2016). Mathematical objects through the lens of two different theoretical perspectives: APOS and OSA. *Educational Studies in Mathematics*, *91*(1), 107–122. https://doi.org/10.1007/s10649-015-9639-6

Fuentealba, C., Badillo, E., & Sánchez-Matamoros, G. (2019). Identificación y caracterización de los subniveles de desarrollo del esquema de derivada. *Enseñanza de las Ciencias, 37*(2), 63–84. https://doi.org/10.5565/rev/ensciencias.2518

Fuentealba, C., Cárcamo, A., Badillo, E., & Sánchez-Matamoros, G. (2023). Análisis de errores en tareas sobre el concepto de derivada: una mirada desde la teoría APOE (Acción, Proceso, Objeto, y Esquema). *Formación Universitaria*, *16*(3), 41–50. https://dx.doi.org/10.4067/s0718-50062023000300041

Fuentealba, C., Sánchez-Matamoros, G., Badillo, E., & Trigueros, M. (2017). Thematization of derivative schema in university students: nuances in constructing relations between a function's successive derivatives. *International Journal of Mathematical Education in Science and Technology*, *48*(3), 374–392. https://doi.org/10.1080/0020739X.2016.1248508

Galindo-Illanes, M. K., Breda, A., Chamorro-Manríquez, D. D., & Alvarado-Martínez, H. A. (2022). Analysis of a teaching learning process of the derivative with the use of ICT oriented to engineering students in Chile. *Eurasia Journal of Mathematics, Science and Technology Education, 18*(7), em2130. https://doi.org/10.29333/ejmste/12162

García, M., Llinares, S., & Sánchez-Matamoros, G. (2011). Characterizing thematized derivative schema by the underlying emergent structures. *International Journal of Science and Mathematics Education*, *9*(5), 1023–1045. https://doi.org/10.1007/s10763-010-9227-2

García, M. M., Romero, I. M., & Gil, F. (2021). Efectos de trabajar con GeoGebra en el aula en la relación afecto-cognición. *Enseñanza de las Ciencias, 39*(3), 177–198. https://doi.org/10.5565/rev/ensciencias.3299

Gavilán-Izquierdo, J. M., García, M., & Martín-Molina, V. (2021). Characterizing the role of technology in mathematics teachers' practices when teaching about the derivative. *Computers in the schools, 38*(1), 36–56. https://doi.org/10.1080/07380569.2021.1882211

Hohenwarter, J., Hohenwarter, M., & Lavicza, Z. (2009). Introducing dynamic mathematics software to secondary school teachers: the case of GeoGebra. *Journal of Computers in Mathematics and Science Teaching*, *28*(2), 135–146. https://www.learntechlib.org/primary/p/30304/

İpek, Z. H., Gözüm, A. İ. C., Papadakis, S., & Kallogiannakis, M. (2023). Educational applications of the ChatGPT AI system: A systematic review research. *Educational Process International Journal*, *12*(3), 26–55. https://doi.org/10.22521/edupij.2023.123.2

Leavy, P. (2017). *Research design: quantitative, qualitative, mixed methods, arts-based, and community-based participatory research approaches.* The Guilford Press. https://doi.org/10.1111/fcsr.12276

Nurwahyu, B., Tinungki, G. M., & Mustangin. (2020). Students' concept image and its impact on reasoning towards the concept of the derivative. *European Journal of Educational Research, 9*(4), 1723–1734. https://doi.org/10.12973/eu-jer.9.4.1723

Oktaç, A., Trigueros, M., & Romo, A. (2019). APOS Theory: Connecting research and teaching. *For the Learning of Mathematics*, *39*(1), 33–37. https://www.jstor.org/stable/26742010

Pecharromán, C. (2013). Naturaleza de los objetos matemáticos: representación y significado. *Enseñanza de las Ciencias*, *31*(3), 121–134. https://doi.org/10.5565/rev/ec/v31n3.931

Rahman, M., Hamid, A., & Asmaun, A. (2025). Enhancing students' strategic competence with GeoGebra and productive disposition: Elevating mathematics learning in the digital age. *International Journal of Education in Mathematics, Science, and Technology*, *13*(4), 930–946. https://doi.org/10.46328/ijemst.4923

Romero, I. M., & García, M. M. (2024). Mathematical attitudes transformation when introducing GeoGebra in the secondary classroom. *Education and Information Technologies*, *29*, 10277–10302. https://doi.org/10.1007/s10639-023-12085-w

Roussou, M., Oliver, M., & Slater, M. (2006). The virtual playground: an educational virtual reality environment for evaluating interactivity and conceptual learning. *Virtual Reality, 10*, 227–240. https://doi.org/10.1007/s10055-006-0035-5

Sánchez-Matamoros, G., García, M., & Llinares, S. (2006). El desarrollo del esquema de derivada. *Enseñanza de las Ciencias*, *24*(1), 85–98. https://doi.org/10.5565/rev/ensciencias.3816

Sánchez-Matamoros, G., García, M., & Llinares, S. (2008). La comprensión de la derivada como objeto de investigación en didáctica de la matemática. *Relime, 11*(2), 267–296.

Santos-Trigo, M., Camacho-Machín, M., & Barrera-Mora, F. (2024). Focusing on foundational Calculus ideas to understand the derivative concept via problem-solving tasks that involve the use of a Dynamic Geometry System. *ZDM Mathematics Education, 56,* 1287–1301. https://doi.org/10.1007/s11858-024-01607-6

Socas, M. (2007). Dificultades y errores en el aprendizaje de las matemáticas. Análisis desde el enfoque lógico semiótico. In M. Camacho, P. Flores, & P. Bolea (Eds.) *Investigación en Educación Matemática XI* (pp.-19–52), La Laguna, Tenerife.

Trigueros, M., Badillo, E., Sánchez-Matamoros, G., & Hernández-Rebollar, L. A. (2024). Contributions to the characterization of the Schema using APOS theory: Graphing with derivative. *ZDM Mathematics Education*, *56,* 1093–1108. https://doi.org/10.1007/s11858-024-01615-6

Urhan, S., Gençaslan, O., & Dost, Ş. (2024). An argumentation experience regarding concepts of calculus with ChatGPT. *Interactive Learning Environments*, *32*(10), 7186–7211. https://doi.org/10.1080/10494820.2024.2308093

Vargas, M. F., Fernández-Plaza, J. A., & Ruiz-Hidalgo, J. F. (2020). Análisis de los argumentos dados por docentes en formación a una tarea sobre derivadas. *PNA, 14*(3), 173–203. https://doi.org/10.30827/pna.v14i3.12229

Vega-Urquieta, M. A., Carrillo-Yañez, J., & Soto-Andrade, J. (2014). Análisis según el modelo cognitivo APOS del aprendizaje construido del concepto de la derivada. *Boletim de Educação Matemática, BOLEMA, 28*(48), 403-429. https://doi.org/10.1590/1980-4415v28n48a20

Ward, E., Inzunsa, S., Hernández, S., & López, F. (2013). Conceptualización y uso de representaciones sobre el concepto de límite en docentes de bachillerato. In A. Berciano, G. Gutiérrez, A. Estepa, & N. Climent (Eds.), *Investigación en educación matemática XVII* (pp. 523–534). SEIEM.

Yao, X. (2024). Opportunity to Learn Function Transformations in the Open-Access GeoGebra Resources. *Digital Experiences in Mathematics Education, 10*, 108–131. https://doi.org/10.1007/s40751-024-00136-x

Yohannes, A., & Chen, H. (2021). GeoGebra in mathematics education: A systematic review of journal articles published from 2010 to 2020. *Interactive Learning Environments, 31*, 5682–5697. https://doi.org/10.1080/10494820.2021.2016861

Published by OmniaScience (www.omniascience.com)

Journal of Technology and Science Education, 2026 (www.jotse.org)